

Stirring Bose-Einstein condensate

Bogdan Damski^{1,2}, Krzysztof Sacha¹, and Jakub Zakrzewski¹

¹ *Instytut Fizyki imienia Mariana Smoluchowskiego, Uniwersytet Jagielloński,
ulica Reymonta 4, PL-30-059 Kraków, Poland*

² *Institut für Theoretische Physik, Universität Hannover, D-30167 Hannover, Germany*
(February 1, 2008)

By shining a tightly focused laser light on the condensate and moving the center of the beam along the spiral line one may stir the condensate and create vortices. It is shown that one can induce rotation of the condensate in the direction opposite to the direction of the stirring.

When a spoon (stick, or another object of a similar shape) is used to stir a liquid, the latter rotates in the direction induced by the stirring object. Is it possible to make the liquid rotating counter-clockwise while stirring it clockwise? The aim of this paper is to show that this counter-intuitive scenario may be realized in a quantum fluid, or more precisely a Bose-Einstein condensate (BEC) when stirring it with the help of a tightly focused laser beam.

Creation of vortices in a BEC and study of their properties has been a subject of quite intensive research for last couple of years (an extensive list of references may be found in [1]). Let us mention that vortices have been created in the BEC experimentally using various methods. Paris group [2] used the rotating anisotropic potential (created by a detuned broad laser beam) to make a direct analog of the rotating bucket experiments [3]. The formation of a vortex is then the result of dynamical instabilities appearing in the course of the experiment [4]. Similar method was used by Ketterle group [5]. Boulder group [6] created vortices in two component condensate, where one fraction was made to rotate with respect to the other by means of the phase engineering technique. The latter technique attempts to create directly the desired vortex state in the condensate.

Various “stirring” propositions have been discussed theoretically [4,7] for creation of vortices. In particular [7] used a localized potential moving on a circular path (with an appropriate smooth turn-on and turn-off of the stirrer). Such a stirring produces the condensate state which may be approximately described as a time dependent combination of the ground state and the vortex state. As time evolves the system undergoes a generalized Rabi oscillation between the ground state and the vortex state.

Similar in spirit is our recent proposition for creation of vortices in a BEC [8]. It relies on an appropriate deformation of a harmonic trapping potential by means of an additional, tightly focused laser beam. The beam approaches the center of the trap moving along a spiral line. The effective interaction of the detuned laser beam with atoms results in an additional effective potential seen by the atomic external degrees of freedom. Neglecting the interaction between atoms, the effective two-dimensional

Hamiltonian, in the frame rotating with the center of the laser beam, reads

$$\hat{H} = -\frac{1}{2} \frac{\partial^2}{\partial x^2} - \frac{1}{2} \frac{\partial^2}{\partial y^2} - \Omega \hat{L}_z + \frac{x^2 + y^2}{2} - u_0 \arctan(|x_0|) \exp\left(-\frac{(x - x_0)^2 + y^2}{2\sigma^2}\right), \quad (1)$$

where Ω is the frequency of the rotation of the laser beam around the center of the trap while $u_0 > 0$, σ and x_0 stand for the parameters of the beam. In Eq. (1) and in the following we work in units defined by the harmonic trap. Changing x_0 from an initial negative value to zero, according to $x_0(t) = x_0(0) + vt$, corresponds to the motion of the laser beam along a spiral line in the laboratory frame. When the beam reaches the center of the trap its intensity is reduced to zero [see $\arctan(|x_0|)$ function in Eq. (1)] and we end up with the harmonic trapping potential only (for details of the method see [8]).

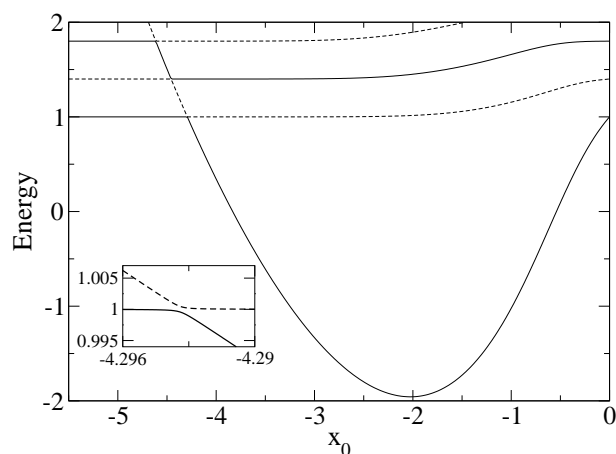


FIG. 1. Energy levels of the Hamiltonian (1) as a function of x_0 for $u_0 = 16$, $\sigma = 0.2$ and $\Omega = 0.6$. The energy levels for $x_0 = 0$ correspond to $L_z = 0$, $L_z = 1$ and $L_z = 2$ (from bottom to top). Note very narrow anti-crossing structures between the neighboring energy levels. The inset shows the anti-crossing between the ground and first excited levels in the enlarged scale.

We have shown, on the other hand, [8] that sweeping the laser beam across the condensate along the spiral line

may serve as an efficient and stable way to create vortices in the system. This can be easily understood by looking at the energy levels of the Hamiltonian (1) for different (fixed) values of x_0 , see Fig. 1. Narrow avoided crossings between neighboring energy levels indicate that starting with the system in the ground state and changing x_0 from some negative value to zero, one may pass the avoided crossing diabatically and end up (with a high efficiency) in the first excited state of the trap that possesses the angular momentum $L_z = 1$. Moreover, the process may be repeated, Fig. 1 suggests that having the system in the first excited state of the harmonic trap with $L_z = 1$ (after a single sweep) it is possible to employ the second similar process and transfer the population to $L_z = 2$ state with a high efficiency.

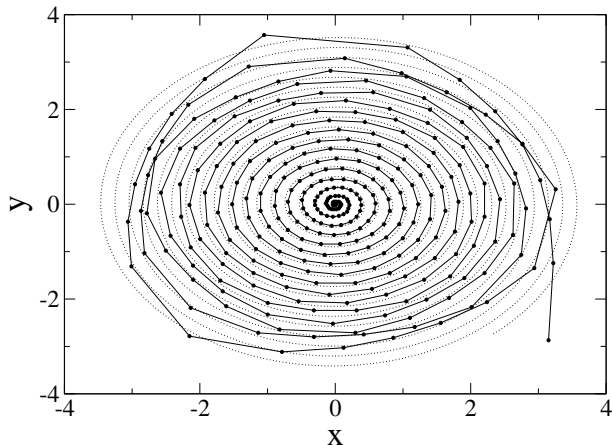


FIG. 2. Trajectories (in the laboratory frame) of the vortex with $n = 1$ (solid line) and of the center of the laser beam (dotted line) from $t = 145$ up to the end of the potential sweeping ($t = 325$). The ground state of the harmonic oscillator was chosen as an initial state and the parameters of the laser beam were $u_0 = 16$, $\sigma = 0.2$, $x_0(0) = -6.5$, $\Omega = 0.6$ and $v = 0.02$. The positions of the vortex (circles on the solid line) were taken with time step that equals 0.54.

To look more quantitatively at the stirring process, we write the wave function of the system, in the hydrodynamical approach [9], as $\Psi(\vec{r}, t) = \sqrt{\rho(\vec{r}, t)} \exp(i\chi(\vec{r}, t))$, where $\rho(\vec{r}, t)$ is the density of a probability fluid. The velocity field is defined as

$$\vec{v} = \vec{\nabla} \chi(\vec{r}, t). \quad (2)$$

The single valuedness of the wave function requires that the circulation of the velocity field Γ_C around any closed contour C is quantized (Feynman-Onsager quantization condition [10])

$$\Gamma_C = \oint_C \vec{v} \cdot d\vec{l} = 2\pi n, \quad (3)$$

where $n = 0, \pm 1, \pm 2, \dots$. The value of n characterizes vortices in the wave function. We say that we have vortex with unit charge at a given point, when calculation

of Γ_C gives $n = 1$ as contour C , encircling that point, shrinks down to this point.

Creation of vortices by our method, which is nothing but a smooth time-dependent modification of the potential, requires a sudden (due to the quantization) appearance of a non-zero circulation. This is necessarily accompanied by an appearance of a singularity in the velocity field. It is interesting to find out how this process occurs since we know that at the beginning of the laser sweep there is no circulation in the velocity field, at the end there is a vortex approximately at the center of the trap. Integrating the time-dependent Schrödinger equation (recall that we discuss noninteracting particles first) we have looked for the wave function modulus minimum and calculated the circulation around small contour encircling it. If n is equal to 1 ± 0.04 (-1 ± 0.04) we assume that vortex (antivortex) with unit charge is located at such a minimum. As almost non-interacting condensates have been realized in laboratories already [11,12], it is perfectly legitimate to consider non-interacting particle case first.

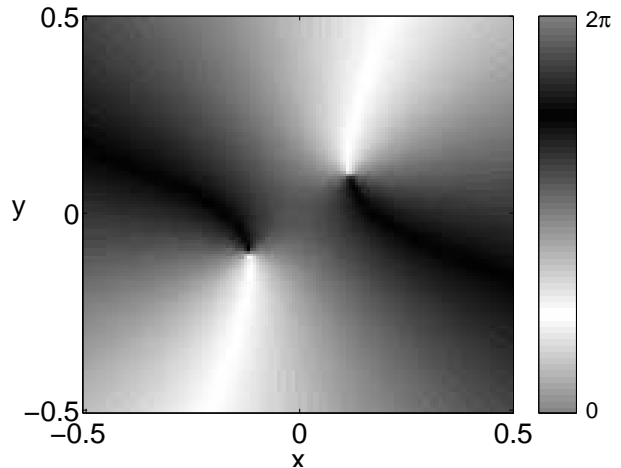


FIG. 3. Plot of the phase of the final wave function after the potential sweeping with the laser beam parameters $u_0 = 16$, $\sigma = 0.2$, $x_0(0) = -6.5$, $\Omega = 0.6$ and $v = 0.02$. The eigenstate of the harmonic oscillator with $L_z = 1$ was chosen as an initial state. Despite the fact that the square overlap of the final wave function on the $L_z = 2$ eigenstate is very high, there is not a single vortex with the topological charge $n = 2$ but two separate vortices with $n = 1$ – see text.

Let us inspect the first sweep of the laser beam through the system initially in the ground state. We have found (compare Fig. 2) that the vortex moves in from the *border* of the trap (i.e. the range of the configuration space where we are able to control the velocity field numerically). Please note that we are able to observe the vortex after some time since the beginning of the simulation. Indeed, it crosses the trap *border* instead of being suddenly created at $t = 145$ —see Fig. 2. The position of the vortex follows quite closely the center of the focused laser beam. At the end of the excitation process the vortex lands very close to the trap center.

Similarly for a second laser sweep aiming at increasing L_z to 2 an additional vortex with the topological charge $n = 1$ comes from the border of the trap along a spiral line (similar to the line depicted in Fig. 2) and collides with the first vortex which, during the whole time evolution, is situated in the vicinity of the trap center. In the numerical implementation the final wave function consists mainly of the eigenstate with $L_z = 2$ (the square overlap on this state is $p_2 \approx 0.9997$). However, there is also a slight contribution from the $L_z = 0$ eigenstate ($p_0 \approx 0.0003$). A simple calculation immediately shows that instead of a single vortex with $n = 2$ we get two separate vortices with $n = 1$ in this case. This observation confirms that vortex with $n = 2$ is unstable. The two vortices are situated symmetrically with respect to the trap center at a distance $2(2p_0/p_2)^{1/4}$. Plot of the phase of the final wave function in the vicinity of the trap center confirms such prediction, see Fig. 3.

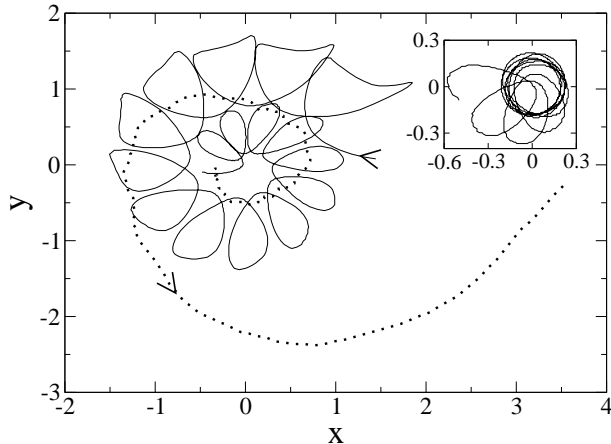


FIG. 4. Trajectories of vortices with the topological charge $n = 1$ (dotted line) and $n = -1$ (solid line) during the potential sweeping. The $L_z = 1$ eigenstate of the harmonic oscillator was chosen as an initial state and the parameters of the laser beam were $u_0 = 16$, $\sigma = 0.2$, $x_0(0) = -6.5$, $\Omega = 0.25$ and $v = 0.02$. The main plot corresponds to $t \in [253, 289]$. After that time the vortex with $n = 1$ topological charge reaches the *border* of the trap and further evolution of the vortex with $n = -1$ up to the end of the potential sweeping ($t = 325$) is shown in the inset. The trajectory of this vortex ends a little off center at $(-0.035, -0.175)$.

Energy levels of the Hamiltonian (1) as a function of x_0 have been calculated in Fig. 1 for $\Omega = 0.6$. For $x_0 = 0$ the ground state corresponds to $L_z = 0$, the first excited state corresponds to $L_z = 1$ and the second one to $L_z = 2$. However, the order can be different if we decrease Ω . Indeed for $\Omega < 1/3$ the second excited state (for $x_0 = 0$) corresponds to $L_z = -1$. It offers an opportunity for the following counterintuitive situation which is of main interest for our study. Suppose, we start with the $L_z = 0$ state. After a potential sweeping we end up with a very high probability in the state with $L_z = 1$ where the rotation of the probability fluid coincides with the rotation of the applied laser beam. Then another,

identical stirring by our “laser spoon” results in probability fluid rotating in the opposite direction (a state with $L_z = -1$)! Needless to say such a situation is quite surprising and no analogy to some process in a classical fluid appears.

The prediction based on Hamiltonian levels behavior can again be tested by a direct integration of the time-dependent Schrödinger equation and indeed the $L_z = -1$ state is excited with very high accuracy. Analyzing the process of such change of the angular momentum from $L_z = 1$ to $L_z = -1$ by looking at the time dependent motion of vortices we find that the vortex with $n = 1$ initially situated at the center moves out to the *border* of the trap while the other vortex (born at the *border*) with an opposite $n = -1$ circulation arrives at the center along a complex trajectory shown in Fig. 4. The latter vortex, before reaching the center, experiences a sequence of collisions with another $n = 1$ vortex that affects its trajectory. Therefore a transition from $n = 1$ to $n = -1$ case is a result of (a bit complicated as seen in Fig. 4) dynamics of vortices.

It remains to be seen whether the counterintuitive stirring scheme is feasible also in the presence of atom-atom interactions since so far we have presented a creation of vortices for a non-interacting BEC. It is known, however, that the stability of vortices may be strongly affected by the atom-atom interactions [13]. To analyze the effect of interactions we have performed numerical integration of the Gross-Pitaevskii equation [14]

$$i \frac{\partial \Psi}{\partial t} = (\hat{H} + g|\Psi|^2)\Psi, \quad (4)$$

with \hat{H} given by (1). The interaction parameter g is proportional to the number of atoms in the system and to the s -wave scattering length. In an experiment, g can be easily of order of thousands but it can be also reduced to a very small value exploring Feshbach resonances [11,12]. In the present work, we have chosen $g = 100$ for the numerical calculations.

If the ground state of the system is chosen as an initial state, applying the potential sweeping allows one to obtain the $L_z = 1$ state with a high efficiency as described in Ref. [8]. We performed such numerical simulation taking $\Omega = 0.1$. Now, we apply the second similar laser sweep on the state obtained after the first one. It creates a vortex with the topological charge $n = -1$ similarly as it takes place for a noninteracting BEC if $\Omega < 1/3$. However, contrary to the linear case, the initial vortex with $n = 1$ does not disappear — the interaction between atoms makes the initial vortex more robust to the perturbation. The vortex with $n = -1$ lands close to the center of the trap while the original $n = 1$ moves to the edge of the trap. In effect the total angular momentum per particle is $\langle \hat{L}_z \rangle = -0.42$ with the dispersion $\sigma_L = \sqrt{\langle \hat{L}_z^2 \rangle - \langle \hat{L}_z \rangle^2} = 1.13$.

The position of vortices may be observed using the interference approach [15]. In left frame of Fig. 5 the

square modulus of the final wave function superimposed with a plane wave traveling vertically in the figure's plane is presented. A vortex-antivortex pair, clearly visible in the figure, can be observed experimentally as the interference technique has been applied in a laboratory already

[16]. The appearance of such a vortex-antivortex pair might be interesting from an experimental point of view, since interactions between such pairs in BEC confined in a harmonic trap, are still an unexplored topic experimentally.

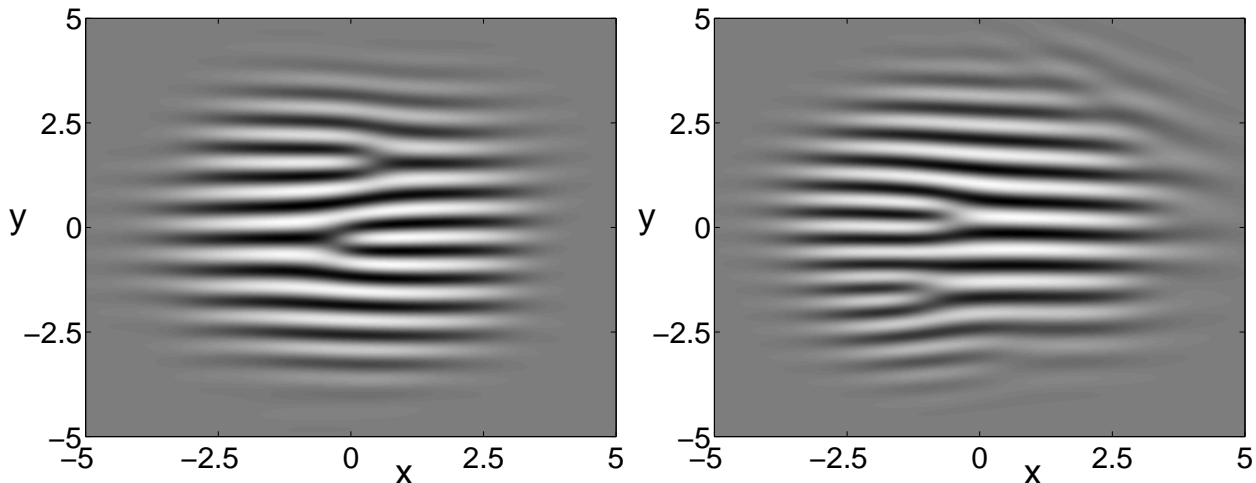


FIG. 5. Interference pictures. Left: a well separated vortex - antivortex pair obtained after a “second” laser sweep through the harmonic potential for $\Omega = 0.1$ and the strength of the effective atom-atom interaction $g = 100$. For $\Omega = 0.25$ one may observe two $n = 1$ vortices (right). Atom-atom interaction ($g = 100$) leads to a big vortex separation - compare with Fig. 3. Parameters of the laser beam are $u_0 = 25$, $\sigma = 0.2$, $x_0(0) = -6.5$, and $v = 0.13$. Time of evolution was equal to 50.

Increasing the frequency Ω of the stirring to $\Omega = 0.25$ we come back to the case of two $n = 1$ vortices discussed previously for noninteracting case. Please note, that now the energy spacing between eigenstates with different value L_z has decreased, so it is possible to have a $L_z = 2$ state as a second excited eigenstate for $\Omega < 1/3$. In the presence of atom-atom interaction ($g = 100$) we again sweep the laser across the condensate twice, first stirring creates a single vortex, a second stirring process adds an additional vortex with the topological charge $n = 1$. This is again in a qualitative agreement with the noninteracting case considered previously. Quantitatively, the final state is characterized by $\langle \hat{L}_z \rangle = 1.69$ with $\sigma_L = 1.92$. The interaction between atoms leads now to a much larger separation between the two vortices, see the right frame in Fig. 5. Indeed, the separation between them is now comparable with the size of the entire condensate (note different scales in Fig. 3 and Fig. 5).

It is interesting to ask what is the critical stirring frequency for a transition from the regime of ‘vortex-antivortex’ to that of ‘vortex-vortex’ production during the second laser sweep. We estimate the critical frequency ω_c as satisfying the following equation:

$$\mu(L_z = -1) + \omega_c = \mu(L_z = 2) - 2\omega_c$$

where $\mu(L_z = -1)$ and $\mu(L_z = 2)$ are chemical potentials of two lowest eigenstates of the time-independent GP equation. The latter are found solving the 2D equation $(\hat{H} + g|\Psi|^2)\Psi = \mu\Psi$, with Hamiltonian $\hat{H} =$

$-\frac{1}{2}\nabla^2 + \frac{1}{2}(x^2 + y^2)$, i.e., the Hamiltonian (1) in the laboratory frame without laser beam. For stirring frequencies Ω lower than ω_c three lowest GP eigenstates, in the frame rotating with stirrer, possess angular momentum $L_z = 0, 1, -1$ while in the case of frequencies higher (but not too high) than ω_c the order is $L_z = 0, 1, 2$. The frequency ω_c is an *upper* bound for the real critical frequency since its definition is based solely on the ordering of eigenstates in the frame rotating with stirrer. Indeed, an efficient transfer requires also that the distance in energy between the level that we would like to populate and the next one should be sufficient to assure adiabaticity, which is by definition not the case when we stir the BEC with $\Omega = \omega_c$. Therefore, one might expect that the optimal realization should require lower frequency, probably in the middle between ω_c and the lowest estimate for a creation of vortex-antivortex pairs (equal to 0). That gives $\omega_c/2$ as a good guess. We expect that the critical frequency should be somewhere between these two estimates, namely between $\omega_c/2$ and ω_c . Calculation for $g = 100$ gives $\omega_c = 0.18$ which interestingly compares with 0.125 ± 0.01 determined from a direct integration of Eq. (4) for $g = 100$, $20 \leq u_0 \leq 25$ and a duration of a single laser sweep between 40 and 60 (compare Fig. 5 and Eq. (1)). Therefore, a numerical calculation gives a value which is higher than $\omega_c/2$ and lower than ω_c , even though the lower bound is just a rough estimate. Similar calculations of upper bounds for the critical frequency (ω_c) yield 0.237 for $g = 30$ and 0.195 for $g = 70$.

Finally, we would like to comment on an influence of the stirring scheme's details on final results in the interacting case. First of all, we have observed that the width σ [compare (1)] should be small, of the order of 0.2; two times bigger widths lead to a significant decrease in the stirring process' efficiency. Secondly, the parameter u_0 has to be high enough, of the order of 20, for an efficient transfer of atoms from the ground state to the vortex state(s). These two conditions provide non-trivial restrictions on laser beam width and intensity, respectively. Thirdly, changes of the switching time within about 20% of a given time scale (≈ 8 periods of harmonic trap for $g = 100$) do not affect the dynamics qualitatively. Further details can be found in [8].

To summarize we have investigated the details of the creation of vortices in BEC when the laser sweep scheme [8] is applied. Especially, we have shown that rotating the probability fluid by means of the "laser spoon" may introduce a circulation with the opposite direction with respect to the steering one.

We are grateful to an anonymous referee for bringing references [4,7] to our attention and several useful hints. Support of KBN under projects 5 P03B 088 21 (K.S. and J.Z.), 2 P03B 124 22 (B.D.) is acknowledged. B. D. is grateful for hospitality extended to him in Hannover during preparation of the final form of manuscript.

- [1] See e.g. the BEC website: <http://amo.phy.gasou.edu/bec.html/bibliography.html>.
- [2] K. W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, Phys. Rev. Lett. **84**, 806 (2000).
- [3] A. J. Leggett, *Topics in Superfluidity and Superconductivity*, in Low Temperature Physics, edited by M. Hoch and R. Lemmer (Springer-Verlag, Berlin, 1992).
- [4] S. Sinha and Y. Castin, Phys. Rev. Lett. **87** 190402 (2001).
- [5] J. R. Abo-Shaeer, C. Raman, J.M. Vogels, and W. Ketterle, Science **292**, 476 (2001).
- [6] M. R. Matthews *et al.*, Phys. Rev. Lett. **83**, 2498 (1999).
- [7] B.M. Caradoc-Davies, R.J. Ballagh and K. Burnett, Phys. Rev. Lett. **83** 895 (1999).
- [8] B. Damski, Z. P. Karkuszewski, K. Sacha, and J. Zakrzewski, Phys. Rev. A **65** 013604-1 (2002).
- [9] S. K. Ghosh, B. M. Deb, Phys. Rep. **92** 1 (1982).
- [10] R. P. Feynman, Statistical Mechanics, W. A. Benjamin, Inc. Advanced Book Program - Reading, Massachusetts.
- [11] L. Khaykovich *et al.* Science **296** 1290 (2002).
- [12] E. A. Donley *et al.* Nature **412** 295 (2001).
- [13] J. J. García-Ripoll, G. Molina-Terriza, V. M. Perez-García, and L. Torner, Phys. Rev. Lett. **87** 140403 (2001).
- [14] F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari Rev. Mod. Phys. **71** 463 (1999).
- [15] E. L. Bolda and D. F. Walls, Phys. Rev. Lett. **81** 5477 (1998).
- [16] S. Innouye *et al.* Phys. Rev. Lett. **87** 080402 (2001).